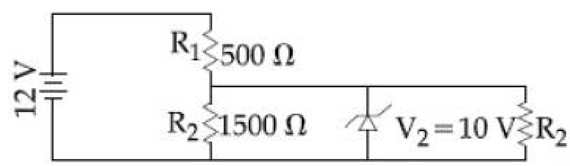
PART –A (PHYSICS)

1. A body is projected at t = 0 with a velocity 10 ms^{-1} at an angle of 60^0 with the horizontal. The radius of curvature of its trajectory at t = 1s is R. Neglecting air resistance and taking acceleration due to gravity g = 10 ms^{-2}, the radius of R is:
(A) 10.3 m (B) 2.8 m
(C) 2.5 m (D) 5.1 m

2. A hydrogen atom, initially in the ground state is excited by absorbing a photon of wavelength 980 Å. The radius of the atom in the excited state, in terms of Bohr radius a_0, will be:
(hc = 12500 eV- Å)
(A) $25a_0$ (B) $9a_0$
(C) $16a_0$ (D) $4a_0$

3. In the given circuit the current through Zener Diode is close to:



(A) 0.0 mA
(B) 6.7 mA
(C) 4.0 mA
(D) 6.0 mA

4. There are two long co – axial solenoids of same length l. The inner and outer coils have radii r_1 and r_2 and number of turns per unit length n_1 and n_2 respectively. The ratio of mutual inductance to the self – inductance of the inner – coil is:
(A) $\frac{n_1}{n_2}$ (B) $\frac{n_2}{n_1}.\frac{r_1}{r_2}$
(C) $\frac{n_2}{n_1}.\frac{r_2^2}{r_1^2}$ (D) $\frac{n_2}{n_1}$

5. A particle undergoing simple harmonic motion has time dependent displacement given by $x(t) = A\sin\frac{\pi t}{90}$. The ratio of kinetic to potential energy o the particle at t = 210s will be
(A) 1/9 (B) 1
(C) 2 (D) 3

6. A satellite is revolving in a circular orbit at a height h from the earth surface, such that h < < R where R is the radius of the earth. Assuming that the effect of earth's atmosphere can be neglected the minimum increase in the speed required so that the satellite could escape from the gravitational field of earth is:
(A) $\sqrt{2gR}$ (B) $\sqrt{gR}$
(C) $\sqrt{\frac{gR}{2}}$ (D) $\sqrt{gR}\left(\sqrt{2}-1\right)$

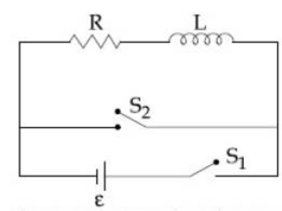
7. The force of interaction between two atoms is given by F = $\alpha\beta \exp\left(-\frac{x^2}{\alpha kt}\right)$; where x is the distance, k is the Boltzmann constant and T is temperature and α and β are two constants. The dimension of β is:

(A) $M^0L^2T^{-4}$
(B) M^2LT^{-4}
(C) MLT^{-2}
(D) $M^2L^2T^{-2}$

8. In a Young's double slit experiment, the path difference, at a certain point on the screen, between two interfering waves is $1/8^{th}$ of wavelength. The ratio of the intensity at this point to that at the centre of a bright fringe is close to:

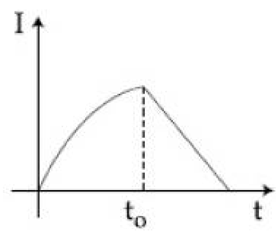
(A) 0.74
(B) 0.85
(C) 0.94
(D) 0.80

9. In the circuit shown,

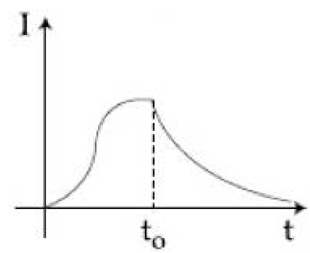


The switch S_1 is close at time t = 0 and the switch S_2 is kept open. At some later time (t_0), the switch S_1 is opened and S_2 is closed. The behaviour of the current I as a function of time 't' is given by:

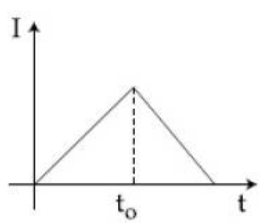
(A)



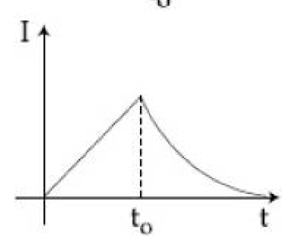
(B)



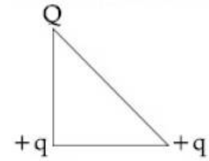
(C)



(D)

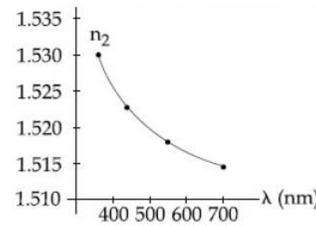


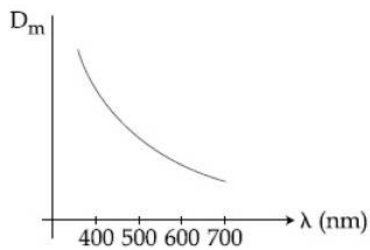
10. Three charges Q, +q and +q are placed at the vertices of a right – angle isosceles triangle as shown below. The net electrostatic energy of the configuration is zero, if the value of Q is:

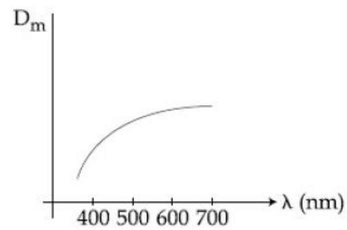


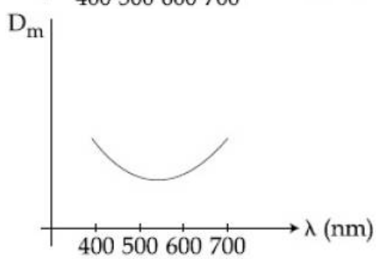
(A) +q
(B) $\frac{-\sqrt{2}q}{\sqrt{2}+1}$
(C) $\frac{-q}{1+\sqrt{2}}$
(D) –2q

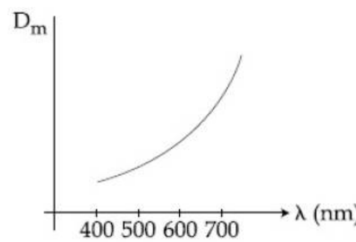
11. The variation of refractive index of a crown glass thin prism with wavelength of the incident light is shown. Which of the following, graph is the correct one, if D_m is the angle of minimum deviation?



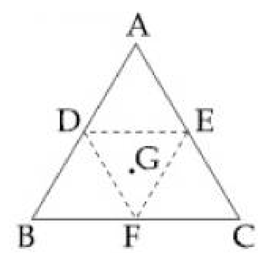
(A)


(B)


(C)


(D)


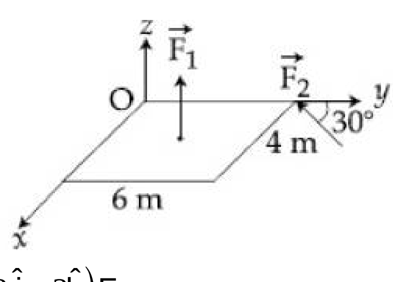
12. The equilateral triangle ABC is cut from a thin solid sheet of wood. (See figure) D, E and F are the mid points of its sides as shown and G is the centre of the triangle. The moment of inertia of the triangle about an axis passing through G and perpendicular to the plane of the triangle is I_0. If the smaller triangle DEF is removed from ABC, the moment of inertia of the remaining figure about the same axis is I. Then:



(A) $I = \frac{15}{16} I_0$ (B) $I = \frac{3}{4} I_0$

(C) $I = \frac{9}{16} I_0$ (D) $I = \frac{I_0}{4}$

13. A slab is subjected to the two forces $\vec{F_1}$ and $\vec{F_2}$ of same magnitude F as shown in the figure. Force $\vec{F_2}$ is in XY – plane while force F_1 acts along z – axis at the point $\left(2\vec{i} + 3\vec{j}\right)$. The moment of these forces about point O will be:



(A) $\left(3\hat{i} + 2\hat{j} + 3\hat{k}\right)F$ (B) $\left(3\hat{i} - 2\hat{j} - 3\hat{k}\right)F$

(C) $\left(3\hat{i} + 2\hat{j} - 3\hat{k}\right)F$ (D) $\left(3\hat{i} + 2\hat{j} + 3\hat{k}\right)F$

14. Ice at -20^0 C is added to 50 g of water at 40^0 C. When the temperature of the mixture reaches 0^o C, it is found that 20 g of ice is still unmelted. The amount of ice added to the water was close to (Specific heat of water = 4.2 J/g/^{0}C)
Heat of fusion of water at 0^oC = 334 J/g)

(A) 50 g
(B) 100 g
(C) 60 g
(D) 40 g

15. A particle is moving along a circular path with a constant speed of 10 ms^{-1}. What is the magnitude of the change in velocity of the particle, when it moves through an angle of 60^{o+} around the centre of the circle?

(A) $10\sqrt{3}$ m / s
(B) 0
(C) $10\sqrt{2}$ m / s
(D) 10 m/s

16. An amplitude modulated signal is given by
$V(t) = 10\left[1 + 0.6\cos\left(2.2\times10^4 t\right)\sin\left(5.5\times10^5\right)t\right]$. Here t is in seconds. The seconds.
The sideband frequencies (in kHz) are. [Given π = 22/7]

(A) 1785 and 1715
(B) 1785 and 1715
(C) 89.25 and 85.75
(D) 892.5 and 857.5

17. An object is at a distance of 20 m from a convex lens of focal length 0.3 m. The lens forms an image of the object. If the object moves away from the lens at a speed of 5 m/s, the speed and direction of the image will be:

(A) 2.26 x 10^{-3} m/s away from the lens
(B) 0.92 x 10^{-3} m/s away from the lens
(C) 3.22 x 10^{-3} m/s towards the lens
(D) 1.16 x 10^{-3} m/s towards the lens

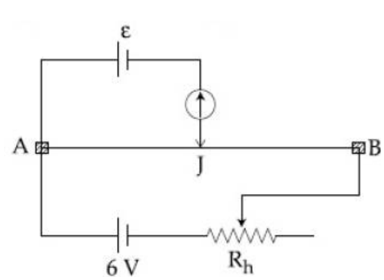
18. A gas mixture consists of 3 moles of oxygen and 5 moles or argon at temperature T. Considering only translational and rotational modes, the total internal energy of the system is:

(A) 15 RT
(B) 12 RT
(C) 4 RT
(D) 20 RT

19. Two equal resistances when connected in series to a battery, consume electric power of 60 W. If these resistances are now connected in parallel combination to the same battery, the electric power consumed will be.

(A) 60 W
(B) 240 W
(C) 120 W
(D) 30 W

20. The resistance of the meter bridge AB in given figure is 4 Ω. With a cell of emf ε = 005 V and rheostat resistance R_h = 2 Ω the null point is obtained at some point J. When the cell is replaced by another one of emf $\varepsilon = \varepsilon_2$ the same null point J is found for R_h = 6 Ω. The emf ε_2 is:



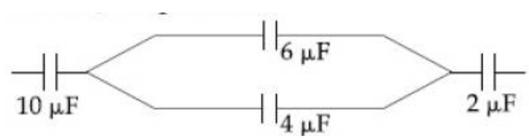
(A) 0.4V
(B) 0.3V
(C) 0.6V
(D) 0.5V

21. An electromagnetic wave of intensity 50 Wm^{-2} enters in a medium of refractive index' n' without any loss . The ratio of the magnitudes of electric fields, and the ratio of the magnitudes of magnetic fields of the wave before and after entering into the medium are respectively. Given by:

(A) $\left(\frac{1}{\sqrt{n}}, \frac{1}{\sqrt{n}}\right)$ (B) $\left(\sqrt{n}, \sqrt{n}\right)$

(C) $\left(\sqrt{n}, \frac{1}{\sqrt{n}}\right)$ (D) $\left(\frac{1}{\sqrt{n}}, \sqrt{n}\right)$

22. In the figure shown below, the charge on the left plate of the μF capacitor is –30μC. The charge on the right place of the 6 μF capacitor is:



(A) –12μC (B) +12μC
(C) –18μC (D) +18μC

23. A rigid diatomic ideal gas undergoes an adiabatic process at room temperature. The rational between temperature and volume for the process is TV^x = constant, then x is:
(A) 3/5 (B) 2/5
(C) 2/3 (D) 5/3

24. A liquid of density ρ is coming out of a hose pipe of radius a with horizontal speed ν and hits a mesh. 50% of the liquid passes through the mesh unaffected. 25% looses all of its momentum and 25% comes back with the same speed. The resultant pressure on the mesh will be:

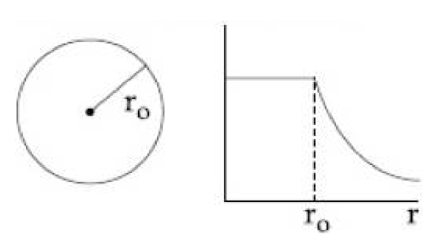
(A) $\frac{1}{4}\rho\upsilon^2$ (B) $\frac{3}{4}\rho\upsilon^2$

(C) $\frac{1}{2}\rho\upsilon^2$ (D) $\rho\upsilon^2$

25. If the deBroglie wavelength of an electron is equal to 10^{-3} times the wavelength of a photon of frequency 6 x10^{14} Hz, then the speed of electron is equal to: (Speed of light = 3 x 10 = 8 = m/s ; Planck's constant = 6.63 x 10^{-34} J.s ; Mass of electron = 9.1 x 10^{-31} kg)

(A) 1.1 x 10^6 m/s (B) 1.7 x 10^6 m/s
(C) 1.8 x 10^6 m/s (D) 1.45 x 10^6 m/s

26. The give graph shown variation (with distance r from centre) of:
(A) Electric field of a uniformly charged sphere
(B) Potential of a uniformly charged spherical shell
(C) Potential of a uniformly charged sphere
(D) Electric field of a uniformly charged spherical shell



27. Equation of travelling wave on a stretched string of linear density 5 g/m is y = 0.03 sin (450 t – 9x) where distance and time are measured in SI united. The tension in the string is:

(A) 10 N (B) 7.5 N
(C) 12.5 N (D) 5 N

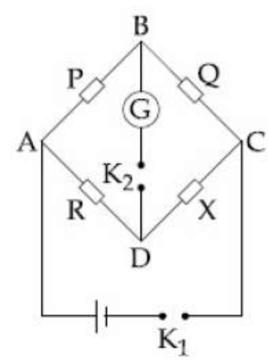
28. In an experiment, electrons are accelerated, from rest, by applying, a voltage of 500 V. Calculate the radius of the path if a magnetic field 100 mT is then applied. [Charge of the electron = 1.6×10^{-19} C Mass of the electron = 9.1×10^{-31} kg]

(A) 7.5×10^{-3} m (B) 7.5×10^{-2} m
(C) 7.5 m (D) 7.5×10^{-4} m

29. A body of mass 1 kg falls freely from a height of 100 m, on a platform of mass 3 kg which is mounted on a spring having spring constant k = 1.25×10^6 N/m. The body sticks to the platform and the spring's maximum compression is found to be x. Given that g = 10 ms^{-2}, the value of x will be close to:

(A) 40 cm (B) 4 cm
(C) 80 cm (D) 8 cm

30. In a Wheatstone bridge (see figure) Resistance P and Q are approximately equal. When R = 400 Ω, the bridge is balanced. On interchanging P and Q, the value of R, for balance is 405 Ω. The value of X is close to:

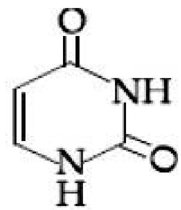


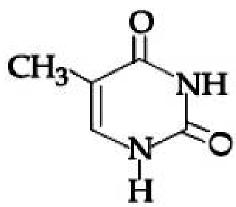
(A) 401.5 ohm
(B) 404.5 ohm
(C) 403.5 ohm
(D) 402.5 ohm

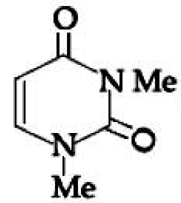
PART –B (CHEMISTRY)

31. Among the following compounds, which one is found in RNA?

(A) 

(B) 

(C) 

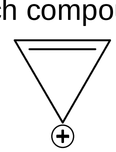
(D) 

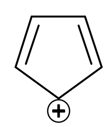
32. The correct match between items I and II is:

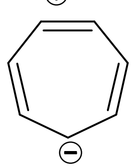
Item – I	Item – II
(Mixture)	(Separation method)
(a) H_2O : Sugar	p. Sublimation
(b) H_2O : Aniline	q. Recrystallization
(c) H_2O : Toluene	r. Stem distillation
	s. Differential extraction

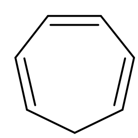
(A) a - d, b – r, c – p
(B) a – q, b – r, c – s
(C) a – r, b – p, c – s
(D) a – q, b – r, c – p

33. Which compounds out of the following is/are not aromatic?

(a) 

(b) 

(c) 

(d) 

(A) b, c and d
(B) c and d
(C) b
(D) a and c

34. A solid having density of 9×10^3 kg m $^{-3}$ forms face centred cubic crystale of edge length $200\sqrt{2}$ pm. What is the molar mass of the solid?
[Avogadro constant $\cong 6 \times 10^{23}$ and mol^{-1}, $\pi \cong 3$]

(A) 0.0432 kg mol^{-1}
(B) 0.0216 kg mol^{-1}
(C) 0.0305 kg mol^{-1}
(D) 0.4320 kg mol^{-1}

35. For the cell $Zn(s)|Zn^{2+}(aq)||M^{x}+(aq)|M(s)$, different half cells and their standard electrode potentials are given below:

$M^{x+}(aq)/M(s)$	$Au^{3+}(aq)/Au(s)$	$Ag^{+}(aq)/Ag(s)$	$Fe^{3+}(aq)/Fe^{2+}(aq)$	$Fe^{2+}(aq)/Fe(s)$
$E^{\circ}_{M^{x+}/M}/(V)$	1.40	0.80	0.77	-0.44

If $E^{0}_{zn^{2}+/zn} = -0.76\,V$, which cathode will given a maximum value of E^{0}_{cell} per electron transferred?

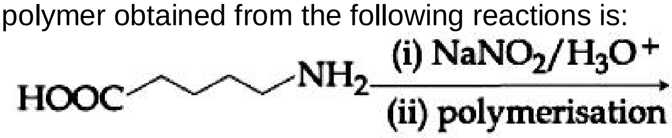
(A) Ag^{+}/Ag
(B) $Fe^{3}+/Fe^{2+}$
(C) $Au^{3}+/Au$
(D) Fe^{2+}/Fe

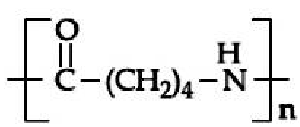
36. The correct match between item I and item II is:

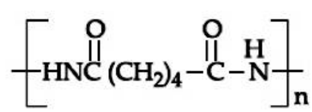
Item – I	Item – II
a. Norethindrone	p. Anti – biotic
b. Ofloxacin	q. Anti – fertility
c. Equanil	r. Hypertention
	s. Analgesics

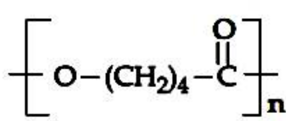
(A) a – q, b – r, c – s
(B) a – q, b – p, c – r
(C) a – r, b – p, c – s
(D) a – r, b – p, c – r

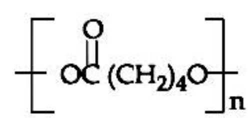
37. The polymer obtained from the following reactions is:



(A) 

(B) 

(C) 

(D) 

38. NaH is an example of:
(A) electron rich hydride
(B) metallic hydride
(C) saline hydride
(D) molecular hydride

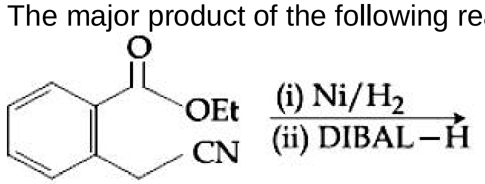
39. The element the usually does NOT show variable oxidation states is:
(A) Cu
(B) Ti
(C) Sc
(D) V

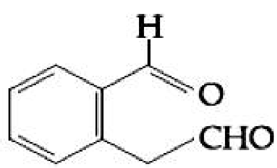
40. An example of solid sol is:
(A) Paint
(B) Gem stones
(C) Butter
(D) Hair cream

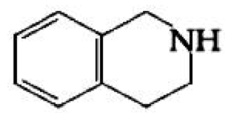
41. The concentration of dissolved oxygen (DO) is cold water can go upto:
(A) 14 ppm
(B) 8 ppm
(C) 10 ppm
(D) 16 ppm

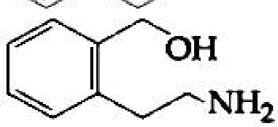
42. The correct statements among (a) to (d) regarding H_2 as a fuel are:
(a) it produces less pollutants than petrol
(b) A cylinder of compressed dihydrogen weighs ~30 times more than a petrol tank producing the same amount of energy.
(c) Dihydrogen is stored in tanks of metal alloys like $NaNi_5$
(d) On combustion, values of energy released per gram of liquid dihydrogen and LPG are 50 and 142 kJ, respectively.
(A) (b) and (d) only
(B) (a) and (d) only
(C) (b), (c) and (d) only
(D) (a), (b) and (c) only

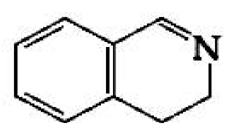
43. The major product of the following reaction is:



(A)


(B)


(C)


(D)


44. The freezing point of diluted milk sample is found to be – 0.2^0C, while it should have been 0.5^0C for pure milk. How much water been added to pure milk to make the diluted sample?
(A) 1 cup of water to 2 cups of pure milk
(B) 3 cups of water to 2 cups of pure milk
(C) 1 cup of water to 3 cups of pure milk
(D) 2 cups of water to 3 cups of pure milk

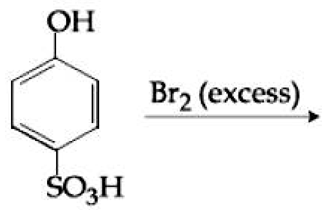
45. If a reaction follows the Arrhenius equation, the plot Ink v 1/(RT) gives straight line with a gradient (–y) unit. The energy required to activate the reactant is:
(A) y/R unit
(B) y unit
(C) yR unit
(D) –y unit

46. A 10 g effervescent tablet containing sodium bicarbonate and oxalic acid releases 0.25 ml of CO_2 at T = 298.15 K and p = 1 bar. If molar volume of CO_2 is 25.0 L under such condition, what is the percentage of sodium bicarbonate in each tablet? [Molar mass of $NaHCO_3$ = 84 g mol^{-1}]
(A) 0.84
(B) 33.6
(C) 16.8
(D) 8.4

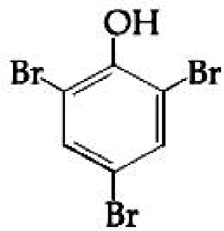
47. The amphoteric hydroxide is:
(A) be $(OH)_2$
(B) $Ca(OH)_2$
(C) $Mg(OH)_2$
(D) $Sr(OH)_2$

48. Peroxyacetyl nitrate (PAN), an eye irritant is produced by:
(A) classical smog
(B) acid rain
(C) organic waste
(D) photochemical smog

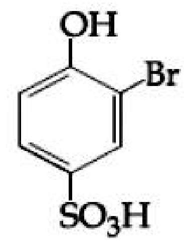
49. The major product of the following reactions is:



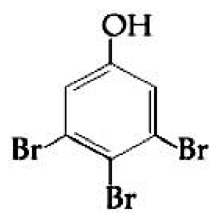
(A)



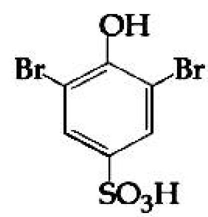
(B)



(C)



(D)



50. An organic compound is estimated through Dumus method and was found to evolve 6 mole of CO_2 4 moles of H_2O and 1 mole of nitrogen gas. The formula of the compound is:

(A) $C_{12}H_8N$
(B) $C_{12}H_8N_2$
(C) $C_6H_8N_2$
(D) C_6H_8N

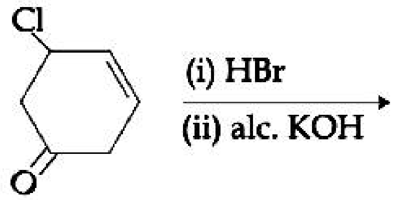
51. Consider the reaction

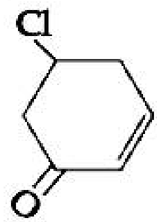
$$N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g)$$

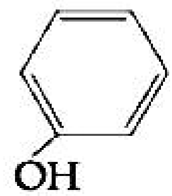
The equilibrium constant of the above reaction is K_3. If pure ammonia is left to dissociate, the partial pressure of ammonia at equilibrium is given by (Assume that $P_{NH_3} << P_{total}$ at equilibrium)

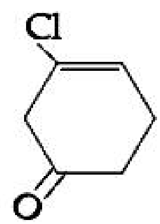
(A) $\frac{3^{3/2} K_p^{1/2} P^2}{16}$
(B) $\frac{K_p^{1/2} P^2}{16}$
(C) $\frac{K_p^{1/2} P^2}{4}$
(D) $\frac{3^{3/2} K_p^{1/2} P^2}{4}$

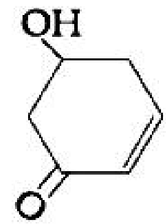
52. The major product of the following reaction is:



(A) 

(B) 

(C) 

(D) 

53. Two blocks of the same metal having same mass and at temperature T_1 and T_2 respectively, are brought in contact with each other and allowed to attain thermal equilibrium at constant pressure. The change in entropy, ΔS, for this process is:

(A) $C_p \ln\left[\frac{(T_1+T_2)^2}{4T_1T_2}\right]$

(B) $2C_p \ln\left[\frac{(T_1+T_2)^{\frac{1}{2}}}{T_1T_2}\right]$

(C) $2C_p \ln\left[\frac{(T_1+T_2)}{4T_1T_2}\right]$

(D) $2C_p \ln\left[\frac{(T_1+T_2)}{2T_1T_2}\right]$

54. Match the metal column I with the coordination compounds/enzymes column II

Column I	Column II
Metals	Coordination compounds/enzymes
a. Co	i. Wilkinson catalyst
b. Zn	ii. Chlorophyll
c. Rh	iii. Vitamin B_{12}
d. Mg	iv. Carbonic anhydrase

(A) a – iii, b – iv, c – i, d - ii
(B) a – i, b – ii, c – iii, d – iv
(C) a – ii, b – i, c – iv, d – iii
(D) a – iv, b – iii, c – i, d – ii

55. For the chemical reaction $X \rightleftharpoons Y$, the standard reaction Gibbs energy depends on temperature T (in K) as

$\Delta_r G^0 \left(\text{in kJ mol}^{-1}\right) = 120 - \frac{3}{8}T$

The major component of the reaction mixture at T is:

(A) Y if T = 300 K
(B) Y if T = 280 K
(C) X if T = 350 K
(D) X if T = 315 K

56. Match the ores (column A) with the metals (column B)

Column A	**Column B**
Ores	Metals
I. Siderite	a. Zinc
II. Kaolinite	b. Copper
III. Malachite	c. Iron
IV. Calamine	d. Aluminium

(A) I – a, II – b, III – c, IV – d
(B) I – c, II – d, III – b, IV – a
(C) I – c, II – d, III – a, IV – b
(D) I – b, II – c, III – d, IV – a

57. Heat treatment of muscular pain involves radiation of wavelength of about 900 nm. Which spectral line of H – atom is suitable for this purpose?

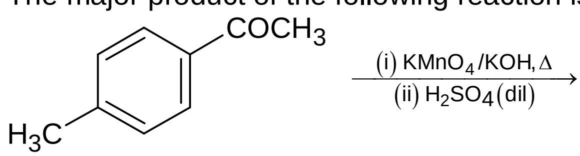
$\left[R_H = 1\times10^5\ \text{cm}^{-1}, h = 6.6\times10^{-34}\ \text{Js}\ c = 3\times10^8\ \text{ms}^{-1}\right]$

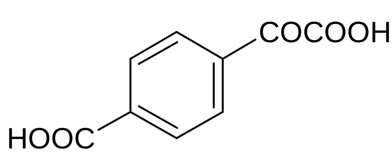
(A) Paschen, $\infty \to 3$
(B) Paschen, $5 \to 3$
(C) Balmer, $\infty \to 2$
(D) Lyman, $\infty \to 1$

58. The chloride that CANNOT get hydrolysed is
(A) $PbCl_4$
(B) CCl_4
(C) $SnCl_4$
(D) $SiCl_4$

59. The correct order of the atomic radii of C, Cs, Al and S is:
(A) C < S < Al < Cs
(B) S < C < Cs < Al
(C) S < C < Al, Cs
(D) C < S < Cs < Al

60. The major product of the following reaction is



(A)


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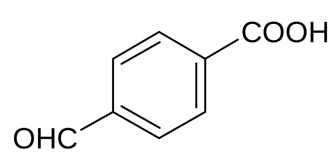
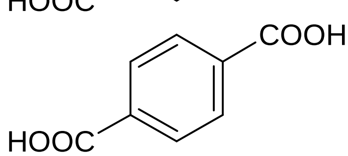
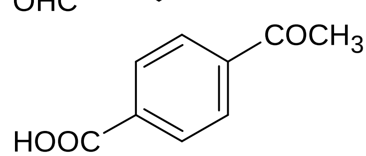


(C)


(D)


PART–C (MATHEMATICS)

61. If the system of linear equations

$2x + 2y + 3z = a$

$3x - y + 5z = b$

$x - 3y + 2z = c$

Where a, b, c are non zero real numbers, has more than one solution, then:

(A) b – c + a = 0 (B) b – c – a = 0

(C) a + b + c = 0 (D) b + c –a = 0

62. Let $f_k(x) = \frac{1}{k}\left(\sin^k x + \cos^k x\right)$ for k = 1, 2, 3, … Then for all $x \in R$, the value of $f_4(x) - f_6(x)$ is equal to:

(A) $\frac{1}{12}$ (B) $\frac{1}{4}$

(C) $\frac{-1}{12}$ (D) $\frac{5}{12}$

63. A square is inscribed in the circle $x^2 + y^2 - 6x + 8y - 103 = 0$ with its sides parallel to the coordinate axes. Then the distance of the vertex of the square which is nearest to the origin is:

(A) 6 (B) $\sqrt{137}$

(C) $\sqrt{41}$ (D) 13

64. If q is false and $p \wedge q \leftrightarrow r$ is true, then which one of the following statements is a tautology?

(A) $(p \vee r) \to (p \wedge r)$ (B) $(p \wedge r) \to (p \vee r)$

(C) $p \wedge r$ (D) $p \vee r$

65. The area (in sq. units) of the region bounded by the curve $x^2 = 4y$ and the straight line $x = 4y - 2$ is:

(A) 5/4 (B) 9/8

(C) 7/8 (D) 3/4

66. The value of the integral $\int_{-2}^{2} \frac{\sin^2 x}{\left[\frac{x}{\pi}\right] + \frac{1}{2}} dx$ (where [x] denotes the greatest integer less than or equal to x) is:

(A) 0 (B) sin 4

(C) 4 (D) 4 – sin 4

67. The outcome of each of 30 items was observed; 10 items gave an outcome $\frac{1}{2}-d$ each, 10 items gave outcome $\frac{1}{2}$ each and the remaining 10 items gave outcome $\frac{1}{2}+d$ each. If the variance of this outcome data is $\frac{4}{3}$ then $|d|$ equals:

(A) $\frac{2}{3}$ (B) 2

(C) $\frac{\sqrt{5}}{2}$ (D) $\sqrt{2}$

68. The plane containing the line $\frac{x-3}{2}=\frac{y+2}{-1}=\frac{z-1}{3}$ and also containing its projection on the plane 2x + 3y – z = 5, contains which one of the following points?

(A) (2, 2, 0) (B) (–2, 2, 2)

(C) (0, –2, 2) (D) (2, 0, –2)

69. Two integers are selected at random from the set {1, 2, …, 11}. Given that the sum of selected numbers is even, the conditional probability that both the numbers are even is:

(A) $\frac{7}{10}$ (B) $\frac{1}{2}$

(C) $\frac{2}{5}$ (D) $\frac{3}{5}$

70. Two circles with equal radii intersecting at the points (0, 1) and (0, –1). The tangent at the point (0, 1) to one of the circles passes through the centre of the other circle. Then the distance between the centres of these circles is:

(A) 1 (B) 2

(C) $2\sqrt{2}$ (D) $\sqrt{2}$

71. The value of r for which ${}^{20}C_r\,{}^{20}C_0+{}^{20}C_{r-1}\,{}^{20}C_1+{}^{20}C_{r-2}\,{}^{20}C_2+\ldots+{}^{20}C_0\,{}^{20}C_r$ is maximum is:

(A) 15 (B) 20

(C) 11 (D) 10

72. If tangents are drawn to the ellipse $x^2+2y^2=2$ at all points on the ellipse other than its four vertices than the mid points of the tangents intercepted between the coordinate axes lie on the curve:

(A) $\frac{1}{4x^2}+\frac{1}{2y^2}=1$ (B) $\frac{x^2}{4}+\frac{y^2}{2}=1$

(C) $\frac{1}{2x^2}+\frac{1}{4y^2}=1$ (D) $\frac{x^2}{2}+\frac{y^2}{4}=1$

73. Let $f(x)=\begin{cases}-1, & -2\le x<0\\ x^2-1, & 0,\le x\le 2\end{cases}$ and $g(x)=|f(x)|+f(|x|)$, Then, in the interval $(-2,2)$, g is:

(A) differentiable at all points (B) not continuous

(C) not differentiable at two points (D) not differentiable at one point

74. If $x\log_e(\log_e x) - x^2 + y^2 = 4\,(y > 0)$, then $\frac{dy}{dx}$ at x = e is equal to:

(A) $\frac{(1+2e)}{2\sqrt{4+e^2}}$ (B) $\frac{(2e-1)}{2\sqrt{4+e^2}}$

(C) $\frac{(1+2e)}{\sqrt{4+e^2}}$ (D) $\frac{e}{\sqrt{4+e^2}}$

75. If $\int \frac{\sqrt{1-x^2}}{x^4}dx = A(x)\left(\sqrt{1-x^2}\right)^m + C$, for a suitable chosen integer m and a function A(x), where C is a constant of integration, then $(A(x))^m$ equals:

(A) $\frac{-1}{27x^9}$ (B) $\frac{-1}{3x^3}$

(C) $\frac{1}{27x^6}$ (D) $\frac{1}{9x^4}$

76. Let [x] denote the greatest integer less than or equal to x. Then:

$$\lim_{x\to 0}\frac{\tan\left(\pi\sin^2 x\right) + \left(|x| - \sin\left(x[x]\right)\right)^2}{x^2}$$

(A) does not exist (B) equals π

(C) equal π + 1 (D) equals 0

77. The maximum value of the function $f(x) = 3x^3 - 18x^2 + 27x - 40$ on the set S = $\{x \in R : x^2 + 30 \le 11x\}$ is

(A) – 122 (B) –222

(C) 122 (D) 222

78. Let $f : R \to R$ be defined by $f(x) = \frac{x}{1+x^2}, x \in R$. Then the range of f is:

(A) $\left[-\frac{1}{2}, \frac{1}{2}\right]$ (B) $R - [-1, 1]$

(C) $R - \left[-\frac{1}{2}, \frac{1}{2}\right]$ (D) $(-1, 1) - \{0\}$

79. The sum of an infinite geometric series with positive terms is 3 and the sum of the cubes of its terms is $\frac{27}{19}$. Then the common ratio of this series is:

(A) $\frac{1}{3}$ (B) $\frac{2}{3}$

(C) $\frac{2}{9}$ (D) $\frac{4}{9}$

80. The straight line x + 2y = 1 meets the coordinate axes at A and B. A circle is drawn through A, B and the origin. Then the sum of perpendicular distances from A and B on the tangent to the circle at the origin is:

(A) $\frac{\sqrt{5}}{2}$ (B) $2\sqrt{5}$

(C) $\frac{\sqrt{5}}{4}$ (D) $4\sqrt{5}$

81. In a triangle, the sum of lengths of two sides is x and the product of the lengths of the same two sides is y. If $x^2 - c^2 = y$, where c is the length of the third side of the triangle, then the circumradius of the triangle is:

(A) $\frac{3}{2}y$ (B) $\frac{c}{\sqrt{3}}$

(C) $\frac{c}{3}$ (D) $\frac{y}{\sqrt{3}}$

82. Equation of a common tangent to the parabola $y^2 = 4x$ and the hyperbola xy = 2 is:

(A) $x + y + 1 = 0$ (B) $x - 2y + 4 = 0$

(C) $x + 2y + 4 = 0$ (D) $4x + 2y + 1 = 0$

83. The sum of the real values of x for which the middle term in the binomial expansion of $\left(\frac{x^3}{3} + \frac{3}{x}\right)^8$ equals 5670 is:

(A) 0 (B) 6

(C) 4 (D) 8

84. Let $a_1, a_2, \ldots, a_{10}$ be a G.P. If $\frac{a_3}{a_1} = 25$, then $\frac{a_9}{a_5}$ equals:

(A) 5^4 (B) $4\left(5^2\right)$

(C) 5^3 (D) $2(5^2)$

85. Let $A = \begin{pmatrix} 0 & 2q & r \\ p & q & -r \\ p & -q & r \end{pmatrix}$. If $AA^T = I_3$, $|P|$ then $|p|$ is:

(A) $\frac{1}{\sqrt{5}}$ (B) $\frac{1}{\sqrt{3}}$

(C) $\frac{1}{\sqrt{2}}$ (D) $\frac{1}{\sqrt{6}}$

86. If y (x) is the solution of the differential equation $\frac{dy}{dx}+\left(\frac{2x+1}{x}\right)y=e^{-2x}, x>0$ where y(1)= $\frac{1}{2}e^{-2}$, then:

(A) $y(\log_e 2)=\log_e 4$
(B) $y(\log_e 2)=\frac{\log_e 2}{4}$
(C) y(x) is decreasing in $\left(\frac{1}{2},1\right)$
(D) y(x) is decreasing in (0, 1)

87. The direction ratios of normal to the plane through the points (0, –1, 0) and (0, 0, 1) and making an angle $\frac{\pi}{4}$ with the plane y – z + 5 = 0

(A) 2, –1, 1
(B) 2, $\sqrt{2}, -\sqrt{2}$
(C) $\sqrt{2}, 1, -1$
(D) $2\sqrt{3}, 1, -1$

88. Let $\vec{a}=\hat{i}+2\hat{j}+4\hat{k}, \vec{b}=\hat{i}+\lambda\hat{j}+4\hat{k}$ and $\vec{c}=2\hat{i}+4\hat{j}+(\lambda^2-1)\hat{k}$ be coplanar vectors. Then the non – zero vector $\vec{a}\times\vec{c}$ is:

(A) $-10\hat{i}-5\hat{j}$
(B) $-14\hat{i}-5\hat{j}$
(C) $-14\hat{i}+5\hat{j}$
(D) $-10\hat{i}+5\hat{j}$

89. If one real root of the quadratic equation $81x^2+kx+256=0$ is cube of the other root, then a value of k is:

(A) – 81
(B) 100
(C) 144
(D) – 300

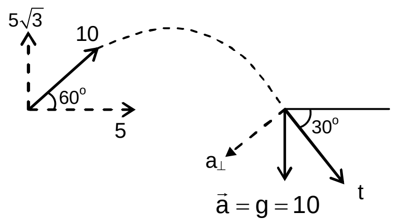
90. Let $\left(-2-\frac{1}{3}i\right)^3=\frac{x+iy}{27}$ $(i=\sqrt{-1})$, where x and y are real numbers, then y – x equals:

(A) 91
(B) – 85
(C) 85
(D) – 91

HINTS AND SOLUTIONS

PART A – PHYSICS

1.



at t = 1

$u_x = 5,\ u_y = 5\sqrt{3}$

$v_y = 5\sqrt{3} - 10 \quad ; \quad v_x = 5$

$\tan\theta = -(2-\sqrt{3}) \quad \Rightarrow \quad \theta = -30°$

$$R = \frac{v^2}{a_\perp} = \frac{10^2}{(10\cos 30^o)}$$

$$= \frac{10}{\sqrt{3}} \times 2 = \frac{20}{\sqrt{3}} m$$

$$\frac{5^2 + (10-5\sqrt{3})^2}{10\cos\theta} = \frac{200-100\sqrt{3}}{10\times 0.965} = 2.8 \text{ m}$$

2. Energy supplied

$$E = \frac{12400}{900} = 12.65 \text{ eV}$$

$\therefore \quad E_n - E_1 = 12.65$

$\Rightarrow \quad (13.6)\left(1 - \frac{1}{n^2}\right) = 12.65$

$\Rightarrow \quad n^2 \approx 14.3$

$\Rightarrow \quad n \approx 4$

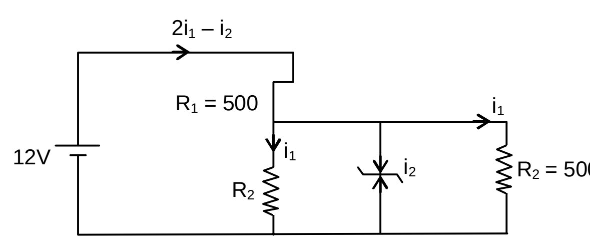
$r \propto n^2$

3. $12 - 500(2i_1 + i_2) - 10 = 0$

$\Rightarrow \quad 2i_1 + i_2 = \frac{2}{500} = \frac{1}{250}$

< i_1 when zenor has break down.

So, $i_2 = 0$.



4. $$\frac{M}{L} = \frac{\mu_0 n n_1 n_2\ n_1^2 \ell}{\mu_0 \pi\ n_1^2 r_1^2 \ell}$$

$$= \frac{n_2}{n_1}$$

5. $K = \frac{1}{2}mv^2 \quad ; \quad U = \frac{1}{2}kx^2 = \frac{1}{2}m^2x^2$

$$\therefore \quad \frac{k}{U} = \frac{v^2}{\omega^2 x^2} = \left(\frac{\cos(wt)}{\sin(wt)}\right)^2$$

$$= \cot^2\left(\frac{\pi}{90} \times 210\right)$$

$$= \cot^2\left(2\pi + \frac{\pi}{3}\right)$$

$$= \left(\frac{1}{\sqrt{3}}\right)^2 = \frac{1}{3}.$$

6. $\Delta v = v_f - v_i$

$$= \sqrt{\frac{2gMe}{R_e}} - \sqrt{\frac{gMe}{R_e}}$$

$$= (\sqrt{2} - 1)\sqrt{gR_e}$$

7. Power of e should be dimensionless.

So, $[\lambda] = (\alpha Tk)$

$\Rightarrow \quad L^2 = [\alpha]\ (ML^2\ T^{-2})$

$\Rightarrow \quad (\alpha) = (M^{-1}\ T^2)$

$\Rightarrow \quad E = \frac{1}{2}KT$

$\Rightarrow \quad (ML^2T^{-2}) \ ; \ (E) = [KT)$

$\Rightarrow \quad (\alpha\beta) = (F)$

$\Rightarrow \quad (M^{-1}\ T^2)\ (\beta) = (MLT^{-2})$

8. $\Delta x = \dfrac{\lambda}{8}$

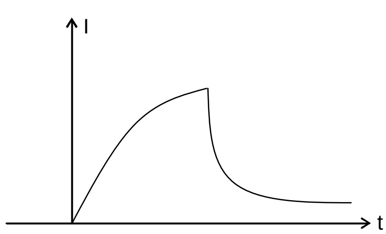
Phase $|\Delta P| = \dfrac{2\pi}{\lambda} \times \dfrac{\lambda}{8} = \dfrac{\pi}{4}$

$\therefore \quad I_{res} = I + I + 2I\ \cos\left(\dfrac{R}{4}\right)$

$$= 2I\left(1 + \frac{1}{\sqrt{2}}\right) = 2I \times 1.7$$

$\therefore \quad \dfrac{I_{res}}{I_{man}} = \dfrac{2I \times 1.7}{4I} = 0.85$

9.



Above is the correct graph for growth and decay of current.

10. $U_{Total} = \dfrac{kQq}{a} + \dfrac{kq^2}{a} + \dfrac{kQq}{a\sqrt{2}} = 0$

$\Rightarrow \quad Q = \dfrac{-q}{\left(1 + \dfrac{1}{\sqrt{2}}\right)}$

11. Prism formula
$D_m = S_m = (n - 1)\,A$ (for thin prism)
So, answer is 1.

12. $I \propto m\ell^2$ (let σ = mass present area)
$\therefore \quad I_1 \propto \ell^4 \qquad \ldots(1)$
and $I_2 \propto \left(\frac{\ell}{2}\right)^4 \qquad \ldots(2)$
So, $I_2 = \frac{I}{16}$
Moment of inertia of remaining sheet = $I - \frac{I}{16}$
$= \frac{15\,I}{16}$

13. $\vec{\tau}_0 = \vec{\tau}_1 + \vec{\tau}_2$
$= (6\hat{i}) \times \left(-\frac{F}{2}\hat{i} + \frac{F\sqrt{3}}{2}\hat{j}\right) + (2\hat{i} + 3\hat{j}) \times (F\hat{k})$
$= -3F\,\hat{k}(-2F\hat{j} + 3F\hat{i})$
$= F(3\hat{i} - 2\hat{j} - 3\hat{k})$

14. $m \times 0.5 \times 20 + (m - 20) \times 80$
$= 50 \times 1 \times 40$
$\Rightarrow$ 90 m – 1600 = 2000
$\Rightarrow$ 90 m = 3600
$\Rightarrow$ m = 40 gm

15. $\Delta\vec{v} = 2v \sin\left(\frac{\theta}{2}\right)$
$= 2 \times 10 \times \sin(30^\circ)$
= 10 m/s

16. $f_{so} = f_c \pm f_m$
$= \frac{\omega_c \pm \omega_m}{2\pi}$
$= \frac{(5.5 \pm 0.22) \times 10^5}{2 \times \frac{22}{7}}$
= 89.25, 85.75

17. $\frac{1}{V} - \frac{1}{-20} = \frac{1}{30}$
$\Rightarrow \quad \frac{1}{V} = \frac{1}{0.30} - \frac{1}{20} = +\frac{200 - 3}{60} = \frac{197}{60}$

$\Rightarrow \quad -\dfrac{dV}{dt\,V^2}+\dfrac{du}{dt\,u^2}=0$

$\Rightarrow \quad \dfrac{dV}{dt}=\dfrac{V^2}{u^2}\,\dfrac{du}{dt} = \left(\dfrac{3}{197}\right)^2\times(-5)=-0.00116\ m/s$

18. $U_{total} = U_{O_2} + U_{Ar}$

$= \dfrac{3\times5\times RT}{2}+\dfrac{5\times3\times RT}{2}$

$= 15\ RT$

19. Assuming constant voltage supply

$P=60=\dfrac{V^2}{R_1+R_2}$ (1)

And $P'=\dfrac{V^2}{R_1}+\dfrac{V^2}{R_2}=\dfrac{2V^2}{R_1} = 4P = 4\times60$

$= 240\ W$

20. $0.5 = \dfrac{6}{(2+\lambda L)}\lambda x$ …(1)

$E_2=\dfrac{6}{(6+\lambda L)}\lambda x$ …(2)

So dividing equation (1) and (2)

$\dfrac{E_2}{0.5}=\dfrac{2+4}{6+4}=\dfrac{3}{5}$

$\Rightarrow \quad E_2 = 0.3$ volt.

21. $\dfrac{E_i}{B_i}=C$ …(1)

$\dfrac{E_f}{B_f}=\dfrac{c}{n}$ …(2)

$\Rightarrow \quad \dfrac{E_iB_f}{E_fB_i}=\dfrac{1}{n}$

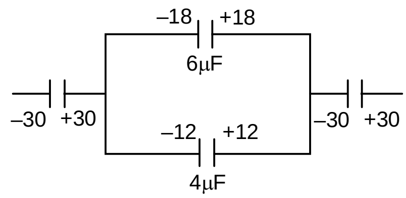
$\Rightarrow \quad \dfrac{E_i}{E_f}=\dfrac{1}{n}\dfrac{B_i}{B_f}$

$\left(\because \quad n=\dfrac{1}{\sqrt{\mu_0 e_r}}\right)$

$\dfrac{1}{\sqrt{n}}:\sqrt{n}$

22. $4V + 6V = 30$

$\Rightarrow \quad V = 3$



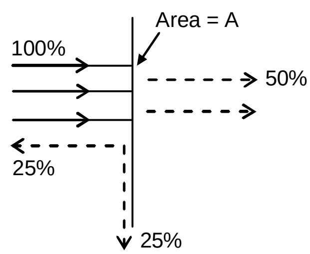
23. Equation of adiabatic process

$TV^{2/f}$ = constant

$\therefore \quad \frac{2}{f} = \frac{2}{5} = x$

24. $F = \frac{1}{4} \times \rho A v^2 + \frac{1}{4} 2\rho a v^2$

$\Rightarrow \quad \frac{F}{A} = \frac{3}{4}\rho A v^2 = \rho$



25. $\lambda_e = \lambda_{photon} \times 10^{-3}$

$\Rightarrow \quad \frac{h}{mv} = \frac{c}{\nu} \times 10^{-3}\,s$

$\Rightarrow \quad v = \frac{h\nu}{mC \times 10^{-3}}$

$= \frac{6.62 \times 10^{-34} \times 6 \times 10^{14}}{9.1 \times 10^{-31} \times 3 \times 10^{8} \times 10^{-3}}$

$= 1.45 \times 10^6$

26. The potential inside a uniformly charged shell is constant, while it decrease hyperbolically outside.

27. $y = 0.03 \sin\left[450\left(t - \frac{9x}{450}\right)\right]$

So, $v = \frac{450}{9} = 50$ m/s

Also, $v = \sqrt{\frac{T}{\lambda}}$

$\Rightarrow \quad 50 = \sqrt{\frac{T}{5 \times 10^{-3}}}$

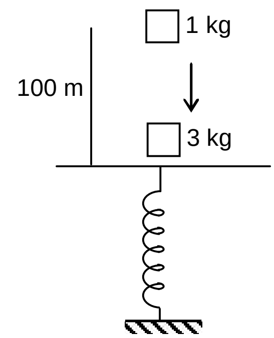
$\Rightarrow \quad T = 2500 \times 5 \times 10^{-3} = 12.5$ N

28. $k_e = \frac{p^2}{2M_e} = 500\,e \qquad \ldots(i)$

& $R = \frac{p}{eB} = \frac{1000\, m_e e}{eB} = \frac{10\,10 \times 9.1 \times 10^{-31}}{1.6 \times 10^{-19}}$

$= 100 \times 7.541 \times 10^{-6}$

29. $v = \sqrt{2 \times 10 \times 100}$
$= 20\sqrt{5}$



COLM $\rightarrow$ $v_p = \dfrac{1 \times 20\sqrt{5}}{4}$
$= 5\sqrt{5}$

COTME $\rightarrow$

$$\frac{1}{2} \times 4 \times (5\sqrt{5})^2 + \frac{1}{2} 1.25 \times 10^6 \left(\frac{30}{k}\right)^2$$

$$= -4 \times 10 \times \left(n - \frac{30}{k}\right) + \frac{1}{2} kn^2$$

$$\Rightarrow \quad 250 + \frac{900}{2 \times 1.25 \times 10^6} = -40x + \frac{1200}{k} + \frac{1}{2} kx^2$$

$\Rightarrow \quad x \simeq 2\text{cm}$

30. $\dfrac{P}{Q} = \dfrac{400}{S}$ ...(1)

and $\dfrac{Q}{P} = \dfrac{405}{5}$ ...(2)

Solving $S^2 = 400 \times 405$

$\Rightarrow \quad S = 402.5\ \Omega$

PART B – CHEMISTRY

31. Fact based

32. Fact based

33. b $\rightarrow$ antiaromatic
d $\rightarrow$ Non aromatic (no cyclic conjugation)

34. Density $= \dfrac{Z \times M}{N_{AV} \times a^3}$

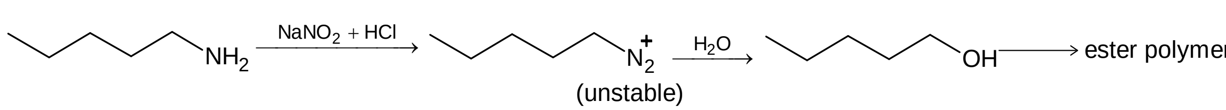
Here Z = 4, a = $200\sqrt{2}$

$N_{AV} = 6.02 \times 10^{23}$, $d = 9 \times 10^3 \dfrac{Kg}{m^3}$

35. $E^o_{Cell} = E^o_{Cathode} - E^o_{Zn^{2+}/Zn}$

Since $Au^{3+} \rightarrow Au$ has maximum value.

36. Fact based.

37 

38. NaH is an example of saline hydride.

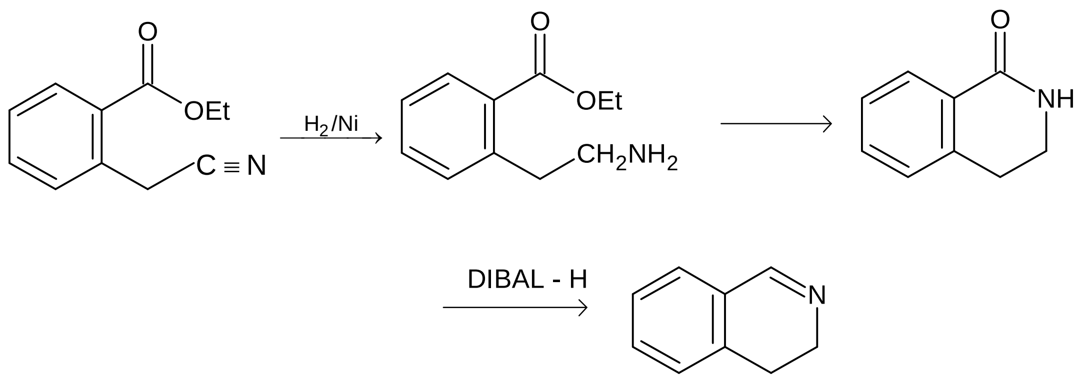
39. Sc^{3+} has noble gas configuration hence only +3 exists.

40. An example of solid sol is gem stones.

41. In cold water, dissolved oxygen can reach a concentration upto 10 ppm

42. Fact based.

43.



44. $0.5\,\alpha\,\dfrac{1}{2}, 0.2\,\alpha\,\dfrac{1}{x}$

Here $\dfrac{0.5}{0.2} = \dfrac{x}{2}$; x = 5, hence 3 cup

45. $\ell nK = -\dfrac{E_a}{RT} + \ell nA$

$\therefore$ Slope = - E_a = -y

46. Let $NaHCO_3$ = x gm
Then, $H_2C_2O_4$ = (10 – x) gm

$\therefore n_{NaHCO_3} = \dfrac{x}{84}$

$2\,NaHCO_3 \rightarrow Na_2CO_3 + H_2O + CO_2$

$\therefore n_{CO_2} = \dfrac{x}{168}$

$n_{H_2C_2O_4} = \left(\dfrac{10-x}{90}\right)$

$H_2C_2O_4 \rightarrow H_2O + CO_2 + CO$

$\therefore n_{CO_2} = \left(\dfrac{10-x}{90}\right)$

Total $CO_2 = \dfrac{x}{168} + \dfrac{10-x}{90} = \dfrac{0.25}{25}$

On solving 'x'

$$\% = \frac{x}{10} \times 100 = 10\,x$$

47. Be – O – H
Both bond has same dissociation energy.

48. Fact based.

49. $-SO_3H$ will be replaced by Br this is called ipso effect.

50. CO_2 = 6 mole, N_1 = 1 mole
C_{atom}= 6, N_{atom} = 2
Hence $C_6H_8N_2$

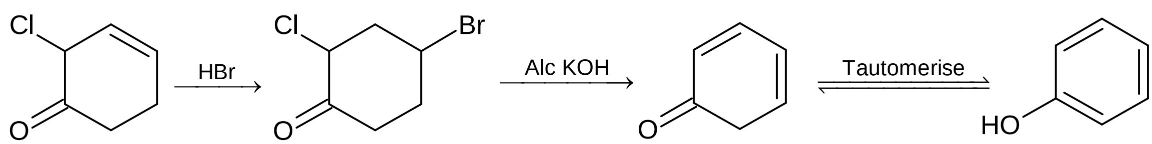
51.
$$N_2 + 3H_2 \rightleftharpoons 2NH_3$$
equ^m x 3x P$_1$

$P_T = 4x$ $K_p = \dfrac{P_1^2}{x \times 27 \times 3}$

$$x = \left(\frac{P}{4}\right)$$

$$P_1 = \sqrt{27x^4K_p}$$

$$\sqrt{27}\left(K_p\right)^{1/2}\left(\frac{P_T}{4}\right)^2 = \frac{3^{3/2}K_p^{1/2}p^2}{16}$$

52.


53. $$\Delta S_{Total} = C_P \ell n \frac{(T_1 + T_2)}{2T_1} + C_P \ell n \frac{(T_1 + T_2)}{2T_2}$$

54. Co $\rightarrow$ Vitamin B_{12}
Zn $\rightarrow$ Carbonic anhydrase
Rh $\rightarrow$ Wilkinson catalyst
Mg $\rightarrow$ Chlorophyll

55. $$\Delta G^o = \left(120 - \frac{3}{8}T\right) = 0$$
Then T = 320 K
Hence T > 320 K Y formed
T < 320 K X formed

56. Siderite $\rightarrow$ Iron
Kaolinite $\rightarrow$ Aluminium
Malachite $\rightarrow$ Copper
Calamine $\rightarrow$ Zinc

57. [i] 900 nm = 9000 $\overset{o}{A}$
It is in far infra red region hence paschen.

58. Central atom has no vacant orbital.

59. On moving down size increases.

60. It is the case of side chain oxidation.

PART C – MATHEMATICS

61. $2x+2y+3z=a \quad (1)$

$3x-y+5z=b \quad (2)$

$x-3y+2z=c \quad (3)$

$(2x+2y+3z)+(x-3y+2z)-(3x-y+5z)=0$

$\Rightarrow a+c-b=0$

62. $F_4(x)=\dfrac{\sin^4 x+\cos^4 x}{4}=\dfrac{1-2\sin^2 x.\cos^2 x}{4}=\dfrac{1}{4}-\dfrac{1}{2}\sin^2 x.\cos^2 x$

$F_6(x)=\dfrac{\sin^6 x+\cos^6 x}{6}=\dfrac{1-3\sin^2 x.\cos^2 x(\sin^2 x+\cos^2 x)}{6}$

$=\dfrac{1}{6}-\dfrac{1}{2}\sin^2 x.\cos^2 x$

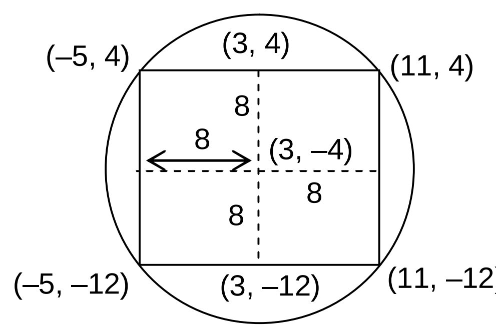
$F_4(x)-f_6(x)=\dfrac{1}{4}-\dfrac{1}{6}=\dfrac{6-4}{24}=\dfrac{2}{24}=\dfrac{1}{12}$

63. Centre (3, –4)

Radius $=\sqrt{9+16+103}=\sqrt{128}=8\sqrt{2}$

$\therefore (-5,\ 4)$ will be nearer to the (0, 0)

$\therefore$ Ans. $\sqrt{5^2+4^2}=\sqrt{25+16}=\sqrt{41}$



64. As $(p\wedge q)\leftrightarrow r$ is true

$\therefore$ If $p\wedge q$ is true and r is true. Or $p\wedge q$ is F and r is F.

As q is false

$p\wedge q$ can not be true

This case is not possible

$\therefore$ Their will be two cases for p, q, r.

T, F, F, or F, F, F respectively

Now check the options.

65. $x^2=4y \quad \text{........}(1)$

$x+2=4y \quad \text{........}(2)$

Solve (1) and (2)

$x+2=x^2$

$\Rightarrow x^2-x-2=0$

$\Rightarrow (x-2)(x+1)=0 \Rightarrow x=2,-1$

$$\text{Area } = \int_{-1}^{2}\left(\frac{x+2}{4}-\frac{x^2}{4}\right)dx$$

$$=\frac{1}{4}\left[\frac{x^2}{2}+2x-\frac{x^3}{3}\right]_{-1}^{2}$$

$$=\frac{1}{4}\left[\left(2+4-\frac{8}{3}\right)-\left(\frac{1}{2}-2+\frac{1}{3}\right)\right]$$

$$=\frac{1}{4}\left[\frac{10}{3}-\frac{3-12+2}{6}\right]=\frac{1}{4}\left[\frac{10}{3}+\frac{7}{6}\right]$$

$$=\frac{1}{4}\times\frac{27}{6}=\frac{9}{8}$$

66. $$\int_{0}^{2}\left(\frac{\sin^2 x}{\frac{1}{2}+\left[\frac{x}{\pi}\right]}+\frac{\sin^2 x}{\frac{1}{2}+\left[-\frac{x}{\pi}\right]}\right)dx$$

$$=\int_{0}^{2}\left(\frac{\sin^2 x}{1+\left[\frac{x}{\pi}\right]}+\frac{\sin^2 x}{\frac{1}{2}-1-\left[\frac{x}{\pi}\right]}\right)dx=\int_{0}^{2}dx=0$$

67. Variance remains some if same number is subtracted from each observation. (subtract 10 from each observation)

$$\therefore\ \frac{10(-d)^2+10(0)^2+10(d)^2}{30}-\left(\frac{10(-d)+10(0)+10(d)}{30}\right)^2=\frac{4}{3}$$

$$\frac{20d^2}{30}=\frac{4}{3}$$

$$\Rightarrow d^2=2$$

$$(d)=\sqrt{2}$$

68. Equation of required plane is

$a(x-3)+b(y+2)+c(z-1)=0$

[as it contains the point (3, –2, 1)]

$2a-b+3c=0$

$2a+3b-c=0$

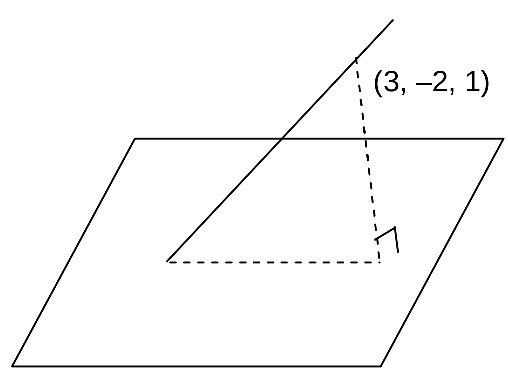
$$\Rightarrow\frac{a}{-8}=\frac{b}{8}=\frac{c}{8}$$

$$\Rightarrow\frac{a}{+1}=\frac{b}{-1}=\frac{c}{1}$$

$\therefore$ equation of plane is

$x-3-y-2-z+1=0$

$\Rightarrow x-y-z=4$



69. $P = \dfrac{\text{both number is even}}{\text{number of ways selecting two numbers such that their sum is even.}}$

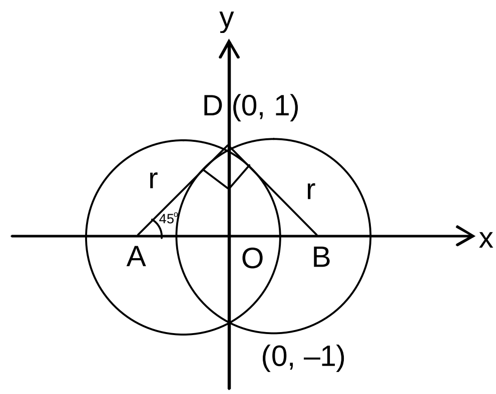
$$= \frac{{}^5C_2}{{}^5C_2 + {}^6C_2} = \frac{10}{10+15} = \frac{10}{25} = \frac{2}{5}$$

70. The two circle will be orthogonal

OD = 1

$\therefore OA = OB = OD = 1$

$\Rightarrow AB = 2$



71. ${}^{20}C_r . {}^{20}C_0 + {}^{20}C_{r-1} . {}^{20}C_1 + ... + {}^{20}C_0 . {}^{20}C_r =$

Selecting r student from 20 boys and 20 girls $= {}^{40}C_r$

${}^{40}C_r$ will be maximum if r = 20.

72. Equation of tangent is

$$\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$$

A is $\left(\dfrac{a}{\cos\theta}, 0\right)$

B is $\left(0, \dfrac{b}{\sin\theta}\right)$

Let P (h, k) is mid point

$$2h = \frac{a}{\cos\theta}$$

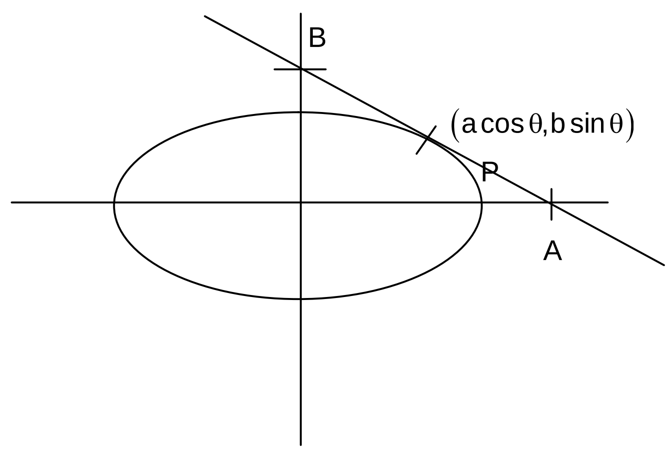
$$2k = \frac{b}{\sin\theta}$$

$$\cos^2\theta + \sin^2\theta = 1$$

$$\Rightarrow \frac{a^2}{4h^2} + \frac{b^2}{4k^2} = 1$$

$$\Rightarrow \frac{2}{4x^2} + \frac{1}{4y^2} = 1$$

$$\Rightarrow \frac{1}{2x^2} + \frac{1}{4y^2} = 1$$



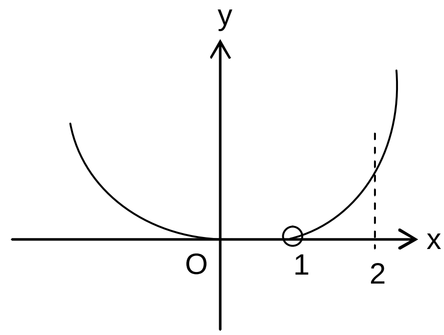
73. $|f(x)| = \begin{cases} 1 & -2 \le x < 0 \\ |x^2 - 1| & 0 \le x \le 2 \end{cases}$

$f(|x|) = \begin{cases} -1 & -2 \le |x| < 0 \\ x^2 - 1 & 0 \le |x| \le 2 \end{cases}$

$\Rightarrow f(|x|) = \{x^2 - 1 \quad -2 \le x \le 2\}$

$g(x) = |f(x)| + f(|x|) : \begin{cases} 1 + x^2 - 1 & -2 \le x < 0 \\ |x^2 - 1| + x^2 - 1 & 0 \le x \le 2 \end{cases}$

$g(x) = \begin{cases} x^2 & -2 < x < 0 \\ 0 & 0 \le x < 1 \\ 2(x^2 - 1) & 1 \le x < 2 \end{cases}$



$\therefore$ $g(x)$ is not differentiable at $x = 1$

74. $x \cdot \frac{1}{\ell n x} \cdot \frac{1}{x} + \log_e(\log_e x) - 2x + 2y \cdot \frac{dy}{dx} = 0$

Put $x = e$

$1 - 2e + 2y\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{2e - 1}{2y}$ (1)

Put x = e in original equation

$O - e^2 + y^2 = 4$

$y^2 = 4 + e^2$

Now put in (i)

$\frac{dy}{dx} = \frac{2e - 1}{2\sqrt{4 + e^2}}$

75. $\int \frac{\sqrt{1 - x^2}}{x^4} dx = \int \frac{x\sqrt{\frac{1}{x^2} - 1}}{x^4} dx = \int \frac{1}{x^3}\sqrt{\frac{1}{x^2} - 1} dx$

Let $\frac{1}{x^2} - 1 = t$

$$-\frac{2}{x^3}dx = dt$$

Integration becomes $-\frac{1}{2}\int \sqrt{t}\,dt = -\frac{1}{2}\frac{(t)^{3/2}}{\frac{3}{2}}$

$$= -\frac{1}{3}\left(\frac{1}{x^2}-1\right)^{3/2}$$

$$= -\frac{1}{3x^3}\left(1-x^2\right)^{3/2}$$

$$= -\frac{1}{3x^3}\left(\sqrt{1-x^2}\right)^3$$

$$\Rightarrow \left(A(x)\right)^3 = \left(-\frac{1}{3x^3}\right)^3 = \frac{1}{27x^9}$$

76. $\underset{x\to 0}{\text{Lt}} \frac{\tan\left(\pi\sin^2 x\right)}{\pi\sin^2 x}.\frac{\pi\sin^2 x}{x^2} + \left(\frac{|x| - \sin\left(x[x]\right)}{|x|}\right)^2$

$1(\pi).(1)^2 + \underset{x\to 0}{\text{Lt}}\left(1-\frac{\sin x[x]}{x[x]}.\frac{x[x]}{|x|}\right)^2$ ....(i)

$$\underset{x\to 0^-}{\text{Lt}} \frac{x[x]}{|x|} = \frac{x}{-x}(-1) = 1$$

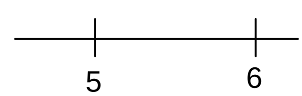
$$\underset{x\to 0^+}{\text{Lt}} \frac{x[x]}{[x]} = \frac{x(0)}{x} = 0$$

Put in equation (i)

$\therefore$ Limit does not exist.

77. $x^2 - 11x + 30 \le 0$

$(x-6)(x-5) \le 0$



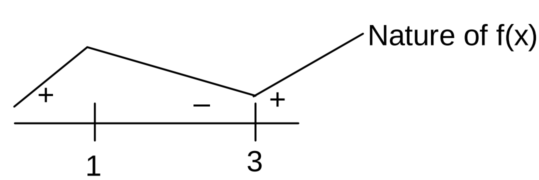
$5 \le x \le 6$

$f(x) = 3x^3 - 18x^2 + 27x - 40$

$f'(x) = 9x^2 - 36x + 27$

$= 9\left(x^2 - 4x + 3\right) = 9(x-3)(x-1)$

$\therefore$ $f(x)$ will be maximum when $x = 6$



$f(6) = 3(6)^3 - 18(6)^2 + 27(6) - 40$

$= 36(18-18) + 162 - 40 = 122$

78. $y = \dfrac{x}{x^2+1}$

$yx^2 - x + y = 0$

$D \ge 0$

$\Rightarrow 1 - 4y^2 \ge 0$

$\Rightarrow y^2 \le \dfrac{1}{4}$

$y \in \left[-\dfrac{1}{2}.\dfrac{1}{2}\right]$

79. $\dfrac{a}{1-r} = 3$

Cube both sides

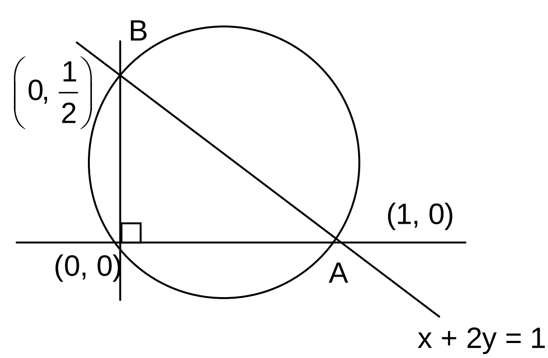
$\dfrac{a^3}{(1-r)^3} = 27$ ……….(1)

and $\dfrac{a^3}{1-r^3} = \dfrac{27}{19}$ ……….(2)

(1) / (2) gives $\dfrac{1-r^3}{(1-r)^3} = 19$

$r = \dfrac{2}{3}$

80. Equation of circle $x(x-1) + \left(y - \dfrac{1}{2}\right)y = 0$



$x^2 + y^2 - x - \dfrac{y}{2} = 0$

Equation of tangent at (0, 0)

$x.0 + y.0 - \dfrac{x+0}{2} - \dfrac{y+0}{2\times 2} = 0$

$2x + y = 0$ ……..(1)

Sum of distance of A and B from Line (i) is

$\dfrac{2}{\sqrt{5}} + \dfrac{\frac{1}{2}}{\sqrt{5}} = \dfrac{5}{2\sqrt{5}} = \dfrac{\sqrt{5}}{2}$

81. $x^2 - c^2 = y$

$(a+b)^2 - c^2 = ab$

$a^2 + b^2 - c^2 = -ab$

$\dfrac{a^2+b^2-c^2}{2ab} = -\dfrac{1}{2}$

$\cos c = -\dfrac{1}{2}$

$c = \frac{2\pi}{3}$

$\sin C = \frac{\sqrt{3}}{2}$

$\frac{c}{\sin c} = 2R \Rightarrow R = \frac{c}{2\sin c} = \frac{c}{\sqrt{3}}$

82. Equation of tangent to parabola $y^2 = 4x$ is $y = mx + \frac{1}{m}$ ………(1)

Now solve it with $xy = 2$

$\left(mx + \frac{1}{m}\right)x = 2$

$\Rightarrow mx^2 + \frac{1}{m}x - 2 = 0$

For common tangent D = 0

$\Rightarrow \frac{1}{m^2} + 8m = 0$

$m = -\frac{1}{2}$

Put it in equation (i) $y = -\frac{1}{2}x - 2$

$\Rightarrow 2x + y + 4 = 0$

83. 5^{th} term will be the middle term.

$t_{4+1} = {}^8C_4\left(\frac{x^3}{3}\right)^4\left(\frac{3}{x}\right)^4 = 5670$

$= {}^8C_4 . x^8 = 5670$

$= \frac{8\times7\times6\times5}{4\times3\times2}x^8 = 5670$

$= x^8 = \frac{567}{7} = 81$

$= x^8 - 81 = 0$

$\Rightarrow$ Real value of $x = \pm\sqrt{3}$

84. $\frac{a_3}{a_1} = \frac{a_1r^2}{a_1} = r^2$

$\Rightarrow r^2 = 25$

Now $\frac{a_9}{a_5} = \frac{a_1r^8}{a_1r^4} = r^4 = (25)^2 = 5^4$

85. A is orthogonal matrix

$\therefore\ 4q^2 + r^2 = p^2 + q^2 + r^2 = 1$ ……..(1)

$p^2 - q^2 - r^2 = 0$ ....(2)

and $2q^2 - r^2 = 0$ ....(3)

Solving (1), (2) and (3)

$p^2 = \frac{1}{2}$

$|p| = \frac{1}{\sqrt{2}}$

86. I.F. $= e^{\int\left(2+\frac{1}{x}\right)dx} = e^{2x}.x$

Solution will be $y\left(xe^{2x}\right) = \int e^{-2x}.xe^{2x}.dx + c$

$xye^{2x} = \frac{x^2}{2} + c$

$1\left(\frac{1}{2}e^{-2}\right).e^2 = \frac{1}{2} + c$ (Put x = 1 and $y = \frac{1}{2}e^{-2}$)

$c = 0$

$\therefore\ y = \frac{x}{2}e^{-2x}$

$\frac{dy}{dx} = \frac{-2xe^{-2x} + e^{-2x}}{2} = \frac{e^{-2x}}{2}(1-2x)$

$\frac{dy}{dx} = 0 \Rightarrow x = \frac{1}{2}$

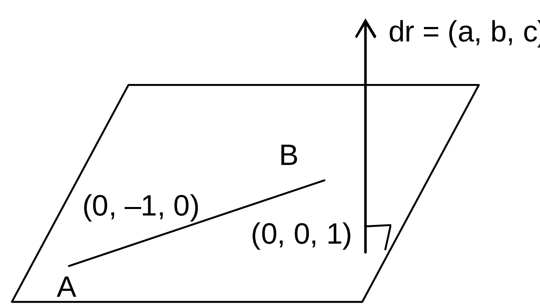
$\frac{dy}{dx} < 0$ if $x \in \left(\frac{1}{2}, 1\right)$

87. d. r. of AB

= (0, 1, 1)

$a.0 + b.1 + c.1 = 0$

$b + c = 0$



$b + c = 0$ ....(i)

$\cos\frac{\pi}{4} = \frac{a.0 + b(1) + c(-1)}{\sqrt{a^2 + b^2 + c^2}.\sqrt{2}}$

$\frac{1}{\sqrt{2}} = \frac{b - c}{\sqrt{a^2 + b^2 + c^2}}.\sqrt{2}$

$= b^2 + c^2 - 2bc$

$\Rightarrow a^2 = -2bc$ ....(ii)

$\Rightarrow a^2 = -2b(-b)$

$a^2 = 2b^2 \quad \Rightarrow \quad a = \pm\sqrt{2}b$

$\frac{a}{\sqrt{2}} = \frac{b}{1}$

From (i) $\frac{a}{\sqrt{2}} = \frac{b}{1} = \frac{c}{-1}$

OR $\frac{a}{-\sqrt{2}} = \frac{b}{1}$

$\Rightarrow \frac{a}{-\sqrt{2}} = \frac{b}{1} = \frac{c}{-1}$ from (i)

$\frac{a}{\sqrt{2}} = \frac{b}{-1} = \frac{c}{1}$

dr. will be $\left(\sqrt{2}, 1, -1\right)$ or $\left(\sqrt{2}, -1, 1\right)$

88. $\left[\vec{a} \quad \vec{b} \quad \vec{c}\right] = 0$

$$\begin{vmatrix} 1 & 2 & 4 \\ 1 & \lambda & 4 \\ 2 & 4 & \lambda^2 - 1 \end{vmatrix}_{R_3 \to R_3 - 2R_1} = 0$$

$$\begin{vmatrix} 1 & 2 & 4 \\ 1 & \lambda & 4 \\ 0 & 0 & \lambda^2 - 9 \end{vmatrix} = 0$$

$\left(\lambda^2 - 9\right)\left(\lambda - 2\right) = 0$

$\Rightarrow \lambda = 2$ OR $\lambda^2 = 5$

$$\vec{a} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 4 \\ 2 & 4 & \lambda^2 - 1 \end{vmatrix}$$

$= \hat{i}\left(2\lambda^2 - 2 - 16\right) - \hat{j}\left(\lambda^2 - 1 - 8\right)$

$= \left(\lambda^2 - 9\right)\left(2\hat{i} - \hat{j}\right) = -5\left(2\hat{i} - \hat{j}\right)$ (put $\lambda = 2$)

89. $\alpha + \alpha^3 = -\frac{K}{81}$ …………(1)

$\alpha^4 = \frac{256}{81}$

$\alpha = \pm\frac{4}{3}$ ………..(2)

From (1) and (2)

$\frac{4}{3} + \frac{64}{27} = \frac{-K}{81}$

$K = -300$

90. $\left(-2-\frac{i}{3}\right)^3=\left(\frac{x+iy}{27}\right)$

$(-1)^3\left(2^3+\frac{i^3}{27}+3(2)\frac{i^2}{9}+3(2)^2.\frac{i}{3}\right)=\frac{x-iy}{27}$

$-\left[8-\frac{i}{27}-\frac{2}{3}+4i\right]=\frac{x+iy}{27}$

$\Rightarrow \frac{x}{27}=-8+\frac{2}{3}$

and $\frac{y}{27}=\frac{1}{27}-4$

$\frac{y-x}{27}=\frac{1}{27}-4+8-\frac{2}{3}$

$=\frac{1+27\times 4-18}{27}$

$=\frac{109-18}{27}$

$=\frac{91}{27}$

$\Rightarrow y-x=91$